JMEPEG (2005) 14:383-387
DOI: 10.1361/10599490523959

©ASM International
1059-9495/$19.00

Viscosity Model for Pure Gases at Atmospheric Conditions

Adango Miadonye and Tara M. McKenna

(Submitted September 24, 2003; in revised form March 21, 2005)

The production processes for petroleum gases use a broad range of simulation packages to reduce the capital, time, and cost associated with actual recovery and pipeline transportation. The viscosity model is an important component of these packages. In this study, two simple-to-use empirical models are presented for predicting the viscosity of petroleum gases: the three-parameter Yaws equation; and the correlation of Miadonye-Clyburn. New values were obtained for the constants in Yaws' equation for various hydrocarbon gases. Alternatively, the Yaws equation has been extended to cover nonhydrocarbon gases, some for the first time, and new values were derived for the constants for these gases. The results obtained with the new constants were compared with the viscosity predictions from both the Yaws and the Miadonye-Clyburn correlations. For four petroleum gases and two nonhydrocarbon gases at temperatures from 100 to 1500 K, the models gave viscosity predictions with overall average absolute deviations of 0.30 and 0.75% for the Yaws correlation with new constants, and 1.17 and 2.7% for the Miadonye-Clyburn correlation for viscosity predictions based on one viscosity value. Both models are simple to incorporate in design and simulation packages, and are accurate within the limits of experimental errors for the viscosities of petroleum gases.

Keywords: gas viscosity correlation, hydrocarbons, transport property of gas, viscosity modeling

1. Introduction

The importance of gases in oil-recovery operations is increasing, as evidenced by the successful use of carbon dioxide, nitrogen, and their mixtures as injection gases in enhanced bitumen and heavy oil recovery. Gases have also been in high demand in other industrial processes, such as in natural gas processing and petrochemical and refinery processes. To meet the high demand by these industries, the gases are delivered through pipelines. The network of gas pipelines within North America has nearly doubled in the past five years. Viscosity is an important property of the gases required in many engineering process designs, ranging from the simulation of gas production at reservoir conditions to the design and operation of pipelines. Several equations for estimating the viscosity of gases have been documented in the literature (Ref 1-3). The most common models are the semiempirical models based on the corresponding states principles, the modified Chapman-Enskog theory, and the hard sphere theory. The principles and limitations of these models have been well documented (Ref 4).

The Chapman-Enskog equation (Ref 4) for viscosity of pure gases is given as:

$$\eta = 26.693 \frac{\sqrt{MT}}{\sigma^2 \Omega_\nu (T^*)} \quad \text{(Eq 1)}$$

$$T^* = \frac{kT}{\varepsilon} \quad \text{(Eq 2)}$$

where η is the viscosity (micropoise), T is the temperature (in degrees Kelvin), and M is the molecular weight (in grams per mole). Other empirical equations used in predicting gas viscosity are based on the theory of Smooth rigid-elastic spheres, Sutherland attracting spheres, and Rough elastic spheres (Ref 3). These models depend on various physical property data and various parameters that are obtained by experimental measurements, thus limiting their accuracy and application in design and simulation packages for onshore and offshore gas production.

Yaws (Ref 5) developed a simple and accurate equation for estimating the viscosities of C_1 to C_4 gaseous compounds:

$$\eta_g = A + BT + CT^2 \quad \text{(Eq 3)}$$

The Yaws method is very simple and allows for easy calculations of viscosity (η_g) in micropoise for a variety of com-

Adango Miadonye and **Tara M. McKenna**, Department of Engineering, Cape Breton University, Sydney, Nova Scotia B1P 6L2, Canada. Contact e-mail: adango_miadonye@capebretonu.ca.

Nomenclature

A, B, C	regression constants in Yaws' equation (Eq 3)
k	Boltzman constant
T	temperature (in degrees Kelvin)
Greek Symbols	
α, β, γ	regression constants for Eq 6
Ω	collision integral for given intermolecular potential function
σ	diameter of spherical molecule, in Å
ε	energy scaling factor
η_g	absolute viscosity of gas

pounds, but it is limited by the temperature range for each compound. The regression constants A, B, and C are given for a large number of gases (Ref 5), and the temperature (in degrees Kelvin) is the only piece of data required to estimate viscosity. In earlier work (Ref 6), a correlation equation was developed (Eq 4) in which only one viscosity value at 25 °C was required to estimate viscosity at any temperature and pressure for natural gas mixtures:

$$\mathrm{Log}\eta_g = \frac{b}{\left[1 + \left(\frac{(t-25)}{298.15}\right)\right]^S} + C + B_o + P^* \exp(d * t) \qquad \text{(Eq 4)}$$

In Eq 4, η is the viscosity Pascals second (Pa-s) and t is the temperature (in degrees centigrade). The values of C, B_o, d, and S were given as 0.7509, $-0.003873*b + 0.04240$, $0.007644*b + 0.01203$, and $-0.05026*b + 0.006011$, respectively. The value of b is derived from the measured viscosity at 25 °C and 1 atm, in the following way:

$$b = \log_{10}\eta_0 - C \qquad \text{(Eq 5)}$$

In this article, Eq 3 and 4 are used, due to their simplicity, to correlate the viscosities of hydrocarbon and nonhydrocarbon gases. In doing this, a larger database with a wider temperature range was used, and a constraint nonlinear regression method was used to obtain new values for the constants in the Yaws equation (i.e., Eq 3). The new values derived for various hydrocarbon gases, and for nonhydrocarbon gases, which were not available in the original Yaws equation, are represented in Eq 6 as the new constants α, β, and γ. These replace the A, B, and C in Yaws' original model. Equation 6 provides a new database of constants for estimating the viscosity of pure gases.

$$\eta_g = \alpha + \beta T + \gamma T^2 \qquad \text{(Eq 6)}$$

where η_g is the viscosity of the pure gas (in micropascals per second), and α, β, and γ are constants for different gases. The results of the viscosity prediction using Eq 6 will be compared with those obtained using Eq 3 and 4.

2. Experimental

The viscosity data of four hydrocarbon and two nonhydrocarbon gases between 110 and 1500 K were obtained from the literature, as indicated in Table 1 (Ref 7-10).

The values for the constants α, β, and γ in Eq 6 were obtained by a constraint nonlinear regression technique on the logarithms of the viscosity data with temperature for each gas, and they are listed in Table 2. The values differ from the values of constants in Yaws' equation (Eq 3), primarily due to the use of a larger database and the method used to correlate the data. To validate the accuracy and applicability of Eq 6, new data on methane from Friend et al. (Ref 11) and carbon dioxide data from Vesovic et al. (Ref 12) were used. These data were not part of the viscosity data used in the development of the new constants for the gases.

Table 1 Type and sources of gas samples used

Gas sample	Total points	Citation source
Methane	134	Trengove and Wakeham (Ref 7)
Ethane	73	Hendl and Vogel (Ref 8)
n-Butane	98	Kuchenmeister and Vogel (Ref 9)
Isobutane	84	Kuchenmeister and Vogel (Ref 10)
Carbon dioxide	161	Trengove and Wakeham (Ref 7)
Nitrogen	14	Nabizadeh and Mayinger (Ref 2)

3. Discussion of Results

A total of 617 viscosity data points, from eight gas samples, including the new methane data from Friend et al. (Ref 11) and carbon dioxide data from Vesovic et al. (Ref 12), was validated with Eq 6 in the temperature range from 100 to 1500 K. The average absolute deviations (AADs) obtained with each equation are summarized in Table 3 for each gas sample. Equation 6 consistently estimated viscosities in the given temperature range with the best accuracy. The percentage of deviation for the estimations on the viscosity of methane are shown in Fig. 1. Equation 6 gave deviations of <2% at high temperatures compared with the higher-percentage errors observed when using Eq 3 and 4 as the temperature increases. Similar trends were observed for all of the gas samples, hydrocarbon and nonhydrocarbon, as illustrated in Fig. 2 to 7. While nitrogen gas is beyond the scope of Eq 3 and 4, Eq 6 gave an AAD of 0.29%.

An excellent match between the predicted and experimental viscosity values was obtained for ethane using Eq 6, as shown in Fig. 2 and 3. Table 4 lists the detailed prediction results obtained with Eq 6 for ethane gas of 9.5×10^{-4} mol/L molar density. With all the hydrocarbon gases used in this study, Eq 3 gave the highest percentage deviations, while Eq 6 yielded the best agreement with the experimental viscosity. These differences in results between the equations were more pro-

Table 2 New values obtained for the constants in Eq 6

Gas sample	α	β	γ
Methane	0.447304	0.038886	-1.1902×10^{-5}
Ethane	−0.788686	0.037143	-1.1634×10^{-5}
n-Butane	−0.261194	0.026812	-3.5840×10^{-6}
Isobutane	−0.23216	0.027598	-5.3228×10^{-6}
Carbon dioxide	0.603234	0.051851	-1.0976×10^{-5}
Nitrogen	4.50847	0.048209	-1.2976×10^{-5}

Table 3 Comparison of the average absolute deviation on gas samples using the three viscosity models

Samples	Temperature range, K	Total data points	AAD, % Eq 3	AAD, % Eq 4	AAD, % Eq 6
Methane	110-1050	134	1.8	2.5	1.1
Ethane	289.8-650.1	73	1.1	0.27	0.07
n-Butane	298.2-625.7	98	0.90	0.30	0.10
Isobutane	297.3-626	84	0.87	0.43	0.07
Carbon dioxide	200-1500	161	2.9	5.6	0.98
Nitrogen	293-1098	14	…	…	0.29
Methane	100-400	31	1.46	0.24	0.13
Carbon dioxide	200-1500	22	2.5	5.7	0.98

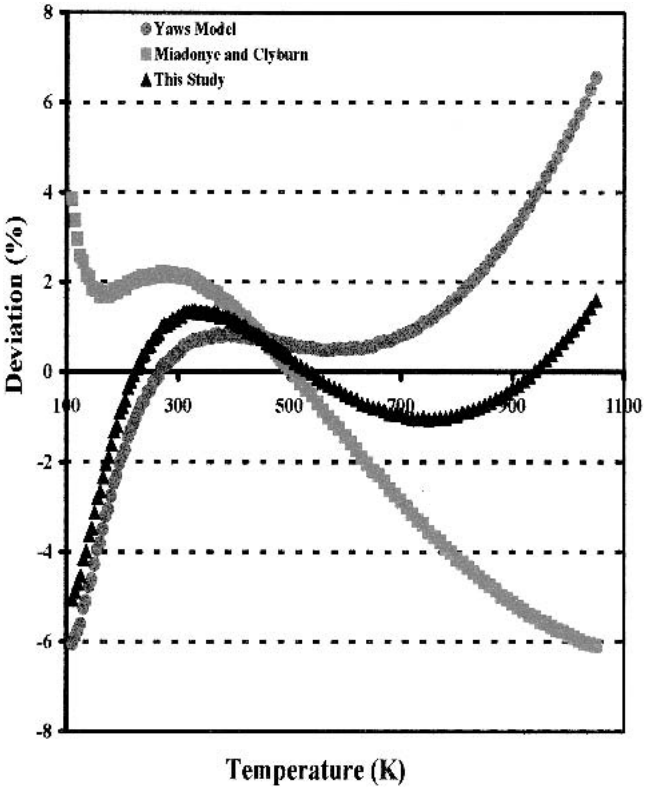

Fig. 1 Deviation with temperatures obtained with the three viscosity models for methane

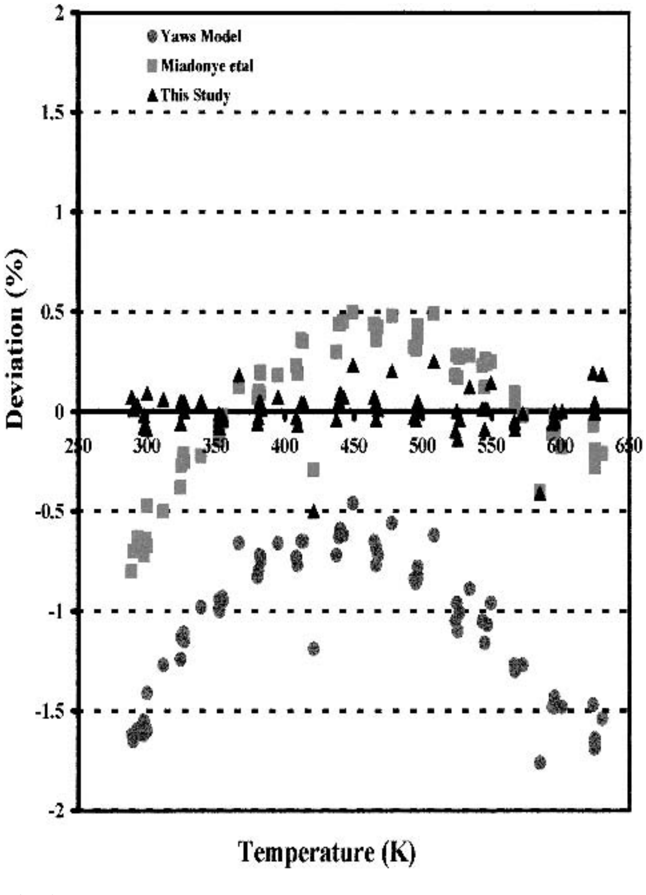

Fig. 2 Deviation with temperatures obtained with the three viscosity models for ethane at various molar densities

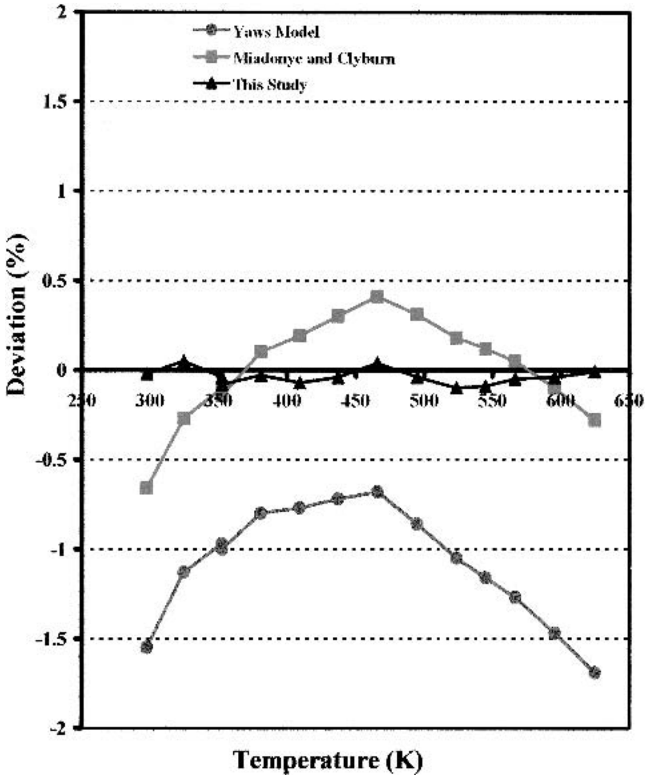

Fig. 3 Deviation with temperatures obtained with the three viscosity models for ethane of a molar density of 31.87×10^{-3} mol/L

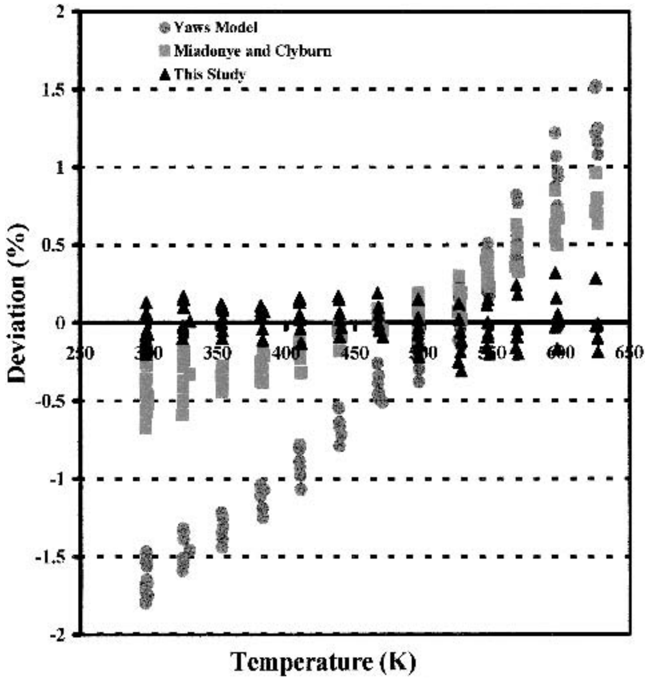

Fig. 4 Deviation with temperatures obtained with the three viscosity models for butane at various molar densities

nounced in the C_4 samples, as shown in Fig. 4 and 5. The comparison indicates that Eq 6 gives far superior viscosity predictions to the Yaws (Ref 5) and Miadonye and Clyburn (Ref 6) correlations. The predictions gave discrepancies that were generally <2% for Eq 3 and 4, and <0.2% for Eq 6.

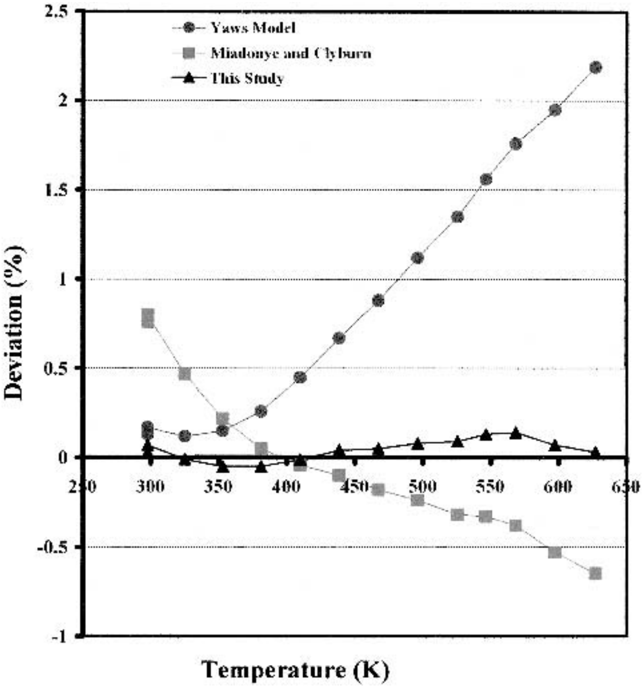

Fig. 5 Deviation with temperatures obtained with the three viscosity models for isobutane of a molar density of 29.55 × 10^{-3} mol/L

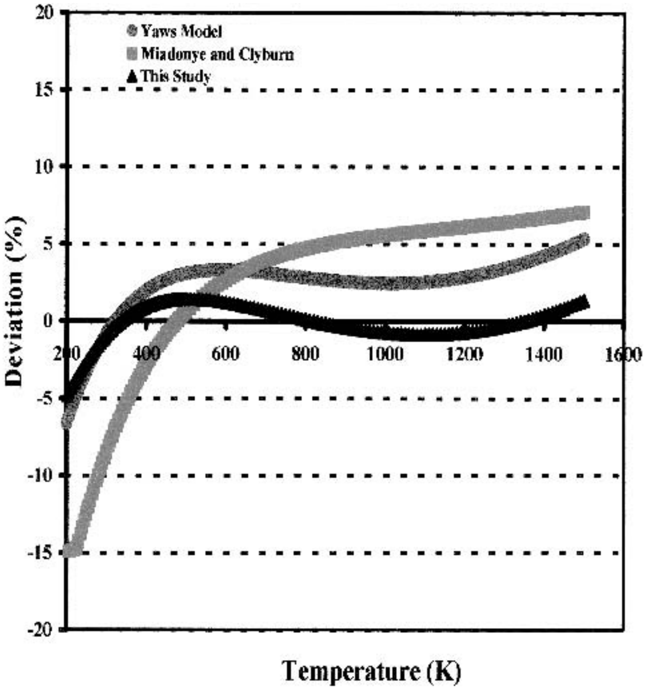

Fig. 6 Deviation with temperatures obtained with the three viscosity models for carbon dioxide

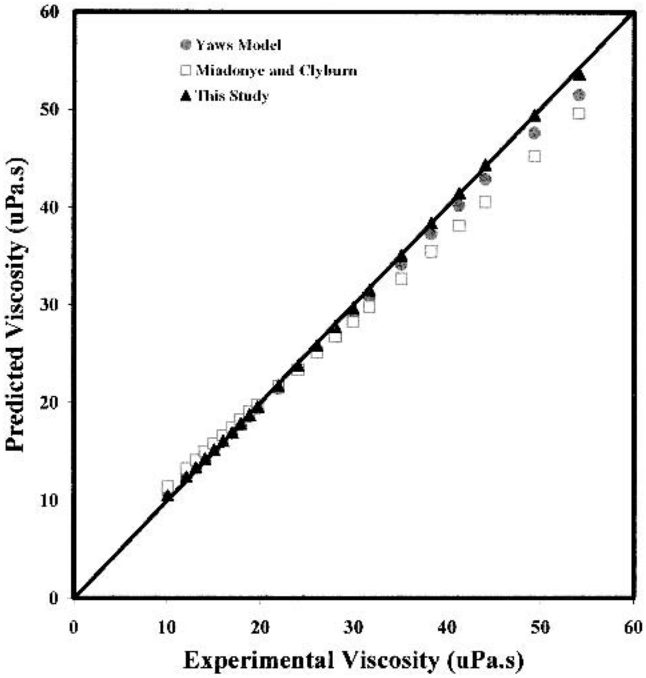

Fig. 7 Comparison of predicted and experimental viscosities for carbon dioxide gas

Table 4 Typical viscosity estimation with Eq 6 at different temperatures for ethane of molar density 9.5 × 10^{-4} mol/L

Temperature, K	Experimental viscosity, μPa/s	Predicted viscosity, μPa/s	Deviation, %
292.88	9.094	9.092	0.03
298.88	9.267	9.273	−0.07
300.06	9.317	9.309	0.09
326.91	10.110	10.110	0.00
354.70	10.920	10.922	−0.02
382.66	11.722	11.721	0.01
411.93	12.543	12.537	0.04
439.23	13.288	13.281	0.05
467.47	14.034	14.032	0.01
496.23	14.785	14.778	0.05
526.58	15.537	15.544	−0.04
545.24	16.006	16.004	0.01
566.49	16.509	16.519	−0.06
595.47	17.204	17.203	0.00
624.80	17.883	17.876	0.04

Note: AAD = 0.04%

Figures 6 and 7 showed the percentage of the deviations on viscosity predictions for carbon dioxide gas. The viscosity predictions with Eq 3 and 6 for carbon dioxide are far more accurate than those obtained with Eq 4. The overall AAD obtained with Eq 6 was 0.98%, which is more accurate by a factor of three to six, respectively, than those obtained with Eq 3 and 4. The detailed results of the viscosity predictions on 22 data points using Eq 6 are given in Table 5. From the results in Table 5 and Fig. 6, it can be seen that Eq 6 gave the best fit for carbon dioxide viscosity in the high-temperature range. The comparison between the estimated and experimental viscosities in Fig. 7 shows that the best match was obtained with Eq 6, as nearly all points fell on the diagonal line. The same cannot be said of the viscosity data points estimated with Eq 3 and 4.

The modified correlation, Eq 6 that was used in this study accommodated the effect of temperature on the viscosity of the gases extremely well. However, it was observed that at temperatures <300 K and >900 K, the correlation showed a tendency to fit gas viscosities at a higher percentage of error. The reason for this behavior might be attributed to the change in the

Table 5 Typical viscosity estimation with Eq 6 at various temperatures for carbon dioxide

Temperature, K	Experimental viscosity, μPa/s	Predicted viscosity, μPa/s	Deviation, %
200	10.05	10.53	−4.82
240	12.07	12.42	−2.86
260	13.06	13.34	−2.16
280	14.05	14.26	−1.50
300	15.02	15.17	−1.00
320	15.98	16.07	−0.57
340	16.93	16.96	−0.20
360	17.87	17.85	0.13
380	18.79	18.72	0.36
400	19.70	19.59	0.57
450	21.90	21.71	0.85
500	24.02	23.78	0.98
550	26.05	25.80	0.96
600	28.00	27.76	0.85
650	29.87	29.67	0.67
700	31.68	31.52	0.50
800	35.09	35.06	0.09
900	38.27	38.38	−0.28
1000	41.26	41.48	−0.53
1100	44.08	44.36	−0.63
1300	49.32	49.46	−0.28
1500	54.13	53.68	0.82

Note: AAD = 0.98%

phases of the gas samples as they approach their condensation and critical temperature regions.

4. Conclusions

The three models presented in this article have correlated the viscosities of various hydrocarbon and nonhydrocarbon gases very well. The new model (Eq 6), and the models of Yaws (Eq 3) and Miadonye-Clyburn (Eq 4) that were used to validate it, yielded accurate results. For the viscosity of hydrocarbon gases, the overall AAD obtained when using Eq 6 was 0.30%, and Eq 3 and 4 gave overall AAD values of 0.88 and 1.17%, respectively. The viscosity estimation on carbon dioxide gas gave an overall AAD of 0.98% using Eq 6, and AAD values of 5.8 and 2.7%, respectively, when using Eq 3 and 4.

Equation 6 gave the best estimation for the viscosity of pure gases for a wider range of temperatures, with an AAD of 0.30%, compared with those the other two models. However, the Miadonye and Clyburn (Ref 6) model uses fewer input data to predict viscosity values. The Miadonye and Clyburn (Ref 6) model is less accurate with an AAD of 1.17% for hydrocarbon gases, but, like the Yaws equation, it is simple to use and requires only one measured viscosity data point at any temperature to estimate gas viscosities at all other temperatures.

Acknowledgments

The authors express their gratitude to the Natural Sciences and Engineering Research Council (NSERC) of Canada, and to Cape Breton University for a Research Policy Committee of Academic Council (RAP) grant. The authors would also like to gratefully acknowledge the assistance of Victoria Whiffen and Ashley Parsons.

References

1. J. Millat, H. Dymond, and C.A. Nieto de Castro, Ed., *Transport Properties of Fluids, Their Correlation, Prediction and Estimation,* Cambridge University Press, Cambridge, U.K., 1996
2. H. Nabizadeh and F. Mayinger, Thermophysical Properties, *Proc. 15th Eur. Conf.,* Wurzburg, Germany, 1999
3. G.C. Maitland, M. Rigby, E.B. Smith, and W.A. Wakeham, *Intermolecular Forces: Their Origin and Determination,* Clarendon Press, Oxford, U.K., 1987
4. S. Chapman and T.G. Cowling, *The Mathematical Theory of Non-Uniform Gases,* 3rd ed., Cambridge University Press, 1970, Chap. 12, p 226-246
5. C.L. Yaws, Viscosity of Gaseous C_1 to C_4 Compounds, *Encyclo. Chem. Proc. Des.,* Vol 62, 1998, p 503-512
6. A. Miadonye and W. Clyburn, Modelling the Viscosity of Natural Gas Mixtures, *Computational Methods and Experimental Measurements,* Vol 11, Esteve et al., Ed., WIT Press, Southhampton, U.K., 2003
7. R.D. Trengove and W.A. Wakeham, The Viscosity of Carbon dioxide, Methane, and Sulfur Hexafluoride in the Limit of Zero Density, *J. Phys. Chem. Ref. Data,* Vol 16 (No. 2), 1987, p 175-187
8. S. Hendl and E. Vogel, The Viscosity of gaseous ethane and its initial density dependence, *Fluid Phase Equilibria,* Vol 76, 1992, p 259-272
9. C. Kuchenmeister and E. Vogel, The Viscosity of Gaseous n-Butane and its initial density dependence, *Int. J. Thermophys.,* Vol 19 (No. 4), 1998, p 1085-1088
10. C. Kuchenmeister and E. Vogel, The Viscosity of Gaseous Iso-butane and Its Initial Density Dependence, *Int. J. Thermophys.,* Vol 21 (No. 2), 2000, p 329-340
11. D.G. Friend, J.F. Ely, and H. Ingham, Thermophysical Porperties of Methane, *J. Phys. Chem. Ref. Data,* Vol 18 (No. 2), 1989, p 583-638
12. V. Vesovic, W. Wakeham, G.A. Olehowy, V. Senger, J.T.R. Watson, and J. Millat, The Transport Porperties of Carbon Dioxide, *J. Phys. Chem. Ref. Data,* Vol 19 (No. 3), 1990, p 763-808